Magnetic Field Sensing Beyond the Standard Quantum Limit Using 10-Spin NOON States

Jonathan A. Jones,1 Steven D. Karlen,2 Joseph Fitzsimons,2,3 Arzhang Ardavan,1 Simon C. Benjamin,2,4 G. Andrew D. Briggs,2 John J. L. Morton1,2*

Quantum entangled states can be very delicate and easily perturbed by their external environment. This sensitivity can be harnessed in measurement technology to create a quantum sensor with a capability of outperforming conventional devices at a fundamental level. We compared the magnetic field sensitivity of a classical (unentangled) system with that of a 10-qubit entangled state, realized by nuclei in a highly symmetric molecule. We observed a 9.4-fold quantum enhancement in the sensitivity to an applied field for the entangled system and show that this spin-based approach can scale favorably as compared with approaches in which qubit loss is prevalent. This result demonstrates a method for practical quantum field sensing technology.

The concept of entanglement, in which coherently correlated (1), has evolved from one of the most startling and controversial outcomes of quantum mechanics to become the enabling principle of emerging technologies such as quantum computation (2) and quantum sensors (3, 4). The use of entangled particles in measurement permits the transcendence of the standard quantum limit in sensitivity, which scales as \(\sqrt{N}\) for \(N\) particles, to the Heisenberg limit, which scales as \(N\). This approach has been applied to optical interferometry by using entangled photons (5) and by using up to six trapped ions for the measurement of magnetic fields and improvements in atomic clocks (6–8). Spin-squeezing has been investigated as an alternative mode of entanglement generation and has been proposed for sensitive phase detection (9) and demonstrated with four \(^{9}\text{Be}\) ions (10).

A single spin will precess in the presence of a magnetic field. In the rotating frame used to describe magnetic resonance, this precession occurs at a rate governed by the detuning \(\delta\) of the magnetic field from resonance (expressed in frequency units), so that the state \(|0\rangle + |1\rangle\) evolves as \(|0\rangle + e^{i\delta t}|1\rangle\) (for clarity, normalization constants are omitted throughout). This principle forms the basis of several kinds of magnetic field sensor, in which the externally applied field \(\delta\) is detected as a phase shift. States possessing many-qubit entanglement can acquire phase at a greater rate and thus offer an enhanced sensitivity to the applied field.

The requirements for constructing the resource of a large number of entangled spins are less severe than those for a complete nuclear magnetic resonance (NMR) quantum computer (11–13). Indeed, rather than striving toward individual addressability of each constituent nuclear spin, global addressing is advantageous in quickly and efficiently growing the state. For example, we considered a star topology with one central spin, \(A\), and \(N\) peripheral \(B\) spins, which cannot be separately addressed (Fig. 1A).

The \(B\) spins cannot be distinguished by means of any NMR observable, and their behavior is well-described by number states, such as those used to describe photon occupation in one of two modes. Many-body entanglement in such states has been referred to as the NOON state (14–16), and has received much attention for its ability to offer quantum-enhanced sensitivity in optical interferometry. We define the spin-NOON state as \(|\psi_{\text{NOON}}\rangle = |N, 0\rangle + |0, N\rangle\), a superposition of the \(N\) spins being all down and all up (this has also been described as a “Schrödinger cat” state, being the superposition of the two most distinct states (17)). Such a spin-NOON state will acquire phase \(e^{i\delta t}\), thus showing an \(N\)-fold increase in the phase accrued for a given \(\delta\) and hence a greater sensitivity to the applied field.

Through single spin-flips, the spin-NOON state may be transformed to “many, some + some, many,” or MSSM states. For example, the state \(|01000\rangle + |10111\rangle\) is one of the five possible \(|\psi_{4112}\rangle\) states. It is convenient to classify these states by the difference in the Hamming weight of the two elements of the superposition, or its lopsidedness \(l\) (that is, \(|\psi_{\text{MSSM}}\rangle\) has \(l = |p - q|\)). In the general case, spins \(A\) and \(B\) have different sensitivities to the applied field, and so the enhancement in magnetic field sensitivity of the

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1Centre for Advanced Electron Spin Resonance (CAESR), Clarendon Laboratory, Oxford University, Oxford OX1 3PU, UK.
2Department of Materials, Oxford University, Oxford OX1 3PH, UK.
3Institute of Quantum Computing, University of Waterloo, Waterloo, ON, N2L 3G1, Canada.
4Centre for Quantum Technologies, National University of Singapore, 3 Science Drive 2, 117543 Singapore.

To whom correspondence should be addressed. E-mail: john.morton@materials.ox.ac.uk
A molecule with a suitable star topology is trimethyl phosphate (TMP) (Fig. 1B), comprising one central $^{31}$P spin and nine identical surrounding $^1$H spins (the intervening O and C nuclei are mostly spin-zero and may be neglected). The NMR spectrum of $^{31}$P is shown in Fig. 1C (red curve) (19). Coupling to the local $^1$H spins shifts the resonance frequency of the $^{31}$P by some amount depending on the total magnetization of the $^1$H. Within the pseudopure state model ($\langle 1I - 13 \rangle$, the lines in the $^{31}$P NMR spectrum can thus be assigned to the following $^1$H states: $|9, 0\rangle$ (0, 0) $|0\rangle$, $|0, 0\rangle$, $\rho_{2,1}, \rho_{2,2}, \ldots, \rho_{1,8}$, $|0, 9\rangle$. Any experimentally accessible “many, some” (MS) state is an equal mixture of the relevant pure states $|M, S\rangle$, where $M$ is the total projection of $^{31}$P which improves with the increasing lopsidedness $l_p$ of the entangled state produced. The intensity has been normalized by using the intensities of the initial spectrum; the residual asymmetry in intensities is an artifact of static field inhomogeneity. The $^{31}$P NMR spectrum of the TMP molecule is shown above for reference. (B) The Fourier transform peak allows an estimate of the effective magnetic detuning from resonance of the $^1$H spins, which improves with the use of higher-$s$ states. Solid and dashed lines represent NMR lines to the left and right of center, respectively. Experimental traces that use the unentangled $^{31}$P or $^1$H spins are shown for comparison. All peaks are scaled to unit intensity.

Fig. 2. Nuclear spin-NOON states demonstrate an entanglement-enhanced sensitivity to an external magnetic field. (A) The Fourier transform of the evolution of each of the 10 NMR lines with respect to $T_{\text{wait}}$ (Fig. 1D) shows an increasing frequency proportional to the increasing lopsidedness $l_p$ of the entangled state produced. The intensity has been normalized by using the intensities of the initial spectrum; the residual asymmetry in intensities is an artifact of static field inhomogeneity. The $^{31}$P NMR spectrum of the TMP molecule is shown above for reference. (B) The Fourier transform peak allows an estimate of the effective magnetic detuning from resonance of the $^1$H spins, which improves with the use of higher-$s$ states. Solid and dashed lines represent NMR lines to the left and right of center, respectively. Experimental traces that use the unentangled $^{31}$P or $^1$H spins are shown for comparison. All peaks are scaled to unit intensity.
equivalent to a canonical Schrödinger cat state that decoheres at the same rate (24, 25). Then, despite being a mixture, 1|MSSM| is nevertheless classified as a Schrödinger cat state of full-sized N within the local decoherence model of (24) (because neither the bit flips nor the mixing inherent in 1|MSSM| alter the rate at which locally independent phases accumulate). If instead we have global decoherence sources, then the effective Schrödinger cat size will correspond to the lopsidedness |M − S| for precisely the reasons of field sensitivity described above.

It is worth contrasting the practical utility of the NOON state approach in two scenarios: in which qubit loss is dominant (such as in optical implementations), and in which phase decoherence is dominant (such as in NMR). Losing even a single photon from a NOON state prevents the phase build up from being measured. Other useful photonic states may have greater inherent robustness (18), but such states have yet to be realized experimentally. As the number of photons in the NOON state is increased, the probability of obtaining a successful measurement decreases exponentially. The sensitivity of the NOON state scales as \(e^{-\log(1 - e)}\) per NOON state, in order to allow a direct comparison of states of varying size. The optimum size of the dashed curves in Fig. 3. The optimum size of an optical NOON state scales as \(e^{-\log(1 - e)}\), beyond which the use of larger entangled states is detrimental to sensitivity. This practical limitation has motivated the development of alternative methods for optical phase sensing in which neither NOON states, nor indeed entangled states of any kind, are employed (22).

Molecular spin-NOON states do not suffer loss in the same manner as optical systems, and the dominant source of error becomes dephasing noise caused by unaccounted-for fields experienced by individual spins. The effect of such noise versus increasing system size can be characterized by using an appropriate measurement strategy. In a noise-free system, the rate at which phase \(\phi\) is acquired by the spin-NOON state would correspond directly to the field strength to be detected. We wish to minimize the variance in this quantity (16) so that

\[
\Delta^2 \left(\frac{\partial \phi}{\partial t}\right) = \frac{\Delta^2 \phi}{N^2} = \frac{1}{N^2 T^2}
\]

Given a fixed time \(T_{\text{tot}}\) to perform the sensor operation, one could make \(M\) separate measurements each of exposure time \(T = T_{\text{tot}}/M - T_{\text{g},r}\) where \(T_{\text{g},r}\) is the gating and measurement time. This strategy will minimize the effects of finite local noise, provided that \(T_{\text{g},r} \ll T_{\text{g}}\). The variance on the mean of \(M\) individual measurements is

\[
\Delta^2 \phi = \frac{1}{M} \left( \frac{1}{N^2 T^2} + \frac{1}{N^2} \sum \Delta^2 h_i \right) = \frac{1}{T_{\text{tot}}} \left( \frac{1}{N^2 T^2} E + \frac{T_{E}}{N} \Delta^2 h \right)
\]

where \(h_i\) is the phase contribution to spin \(i\) from local fields. For any nonzero \(\Delta^2 h\), minimizing this quantity will yield \(T_{E} \propto N^{-1/2}\) resulting in \(\Delta^2 \phi \propto N^{-3/2}\). The sensitivity of the system thus limits to \(N^{3/4}\). Provided that the measurements can be made on a short time scale as compared with the decoherence time of the spin-NOON state (\(\propto N^{-1/2}\)), creating larger entangled states will produce greater sensitivity.

In addition to demonstrating how an enhanced sensitivity to magnetic fields can be achieved by using entanglement in nuclear spins, this work represents progress toward the realization of “spin amplification” schemes, which use a bath of spin states to measure the state of a spin for the purposes of single-spin detection (20). Analogous to the way in which photon loss poses a limitation to the extent of the resource (photon number), which can be called up for entanglement-enhanced measurement, a weak thermal polarization restricts the effectiveness of this demonstration for practical magnetometry. Fortunately, the approach described here is readily applicable to electron spins, which can offer a high degree of polarization at experimentally accessible magnetic fields and temperatures. Furthermore, dynamic nuclear polarization, which is already employed in several methods for magnetic field sensing by using nuclear spins (26) or algorithmic cooling (27, 28), could be applied here to yield improvements over currently achievable sensitivity.

References and Notes
19. Materials and methods are available as supporting material on Science Online.
21. All states of the form 1|MSSM| (defined in Eq. 1) contain one full entbit of entanglement as described in more detail in (19).
25. The second approach of (24), based on distillation, does not immediately generalize from their pure states to our mixed states: Under the constraint of single-qubit operations one cannot produce—with unit probability—
29. We thank P. Kok for helpful discussions. This research is supported by the Engineering and Physical Sciences Research Council through the Quantum Information Processing Interdisciplinary Research Collaboration (www.qipirc.org) (GRS8217/01) and CAESR (EPD048559/1). J.J.L.M. acknowledges St. John’s College, Oxford, J.J.L.M. S.C.B., and A.A. acknowledge support from the Royal Society.

Supporting Online Material
www.sciencemag.org/cgi/content/full/1170730/DC1
Materials and Methods
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References
9 January 2009; accepted 25 March 2009
Published online 23 April 2009
10.1126/science.1170730
Include this information when citing this paper.